

Observational Constraints on the Topology (Global Geometry) of the Universe

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Abstract The Universe is a physical object. Physical objects have shapes and sizes. General relativity is insufficient to describe the global shape and size of the Universe: the Hilbert-Einstein equations only treat limiting quantities towards an arbitrary point. Empirical work on measuring the shape and size of the Universe (formally: the “3-manifold of the spatial hypersurface at constant cosmological time”, and, e.g. the “injectivity diameter” respectively) has progressed significantly in the late 1980’s and the 1990’s, using observational catalogues of galaxy clusters, of quasars and of the microwave background, though the analyses are still hindered by simplifying (and often observationally unsupported) assumptions. A review of the different observational strategies and claimed constraints was presented at the meeting.

1 Introduction

The Universe is a physical object. Physical objects have shapes and sizes.

So, a major goal of observational cosmology is to measure the shape and size of the Universe (within appropriate mathematical theory relating to geometry), or else to convincingly show that these are unmeasurable. The alternative, to suppose that the Universe is a spiritual object, without a shape or size, is not part of the domain of science.

1.1 Relativity and geometry

General relativity relates local geometry to the physical content of the Universe. However, it is insufficient to describe the global shape and size of the Universe: the Hilbert-Einstein equations only treat limiting quantities towards an arbitrary point, i.e. they are local.

An extension of general relativity, for example, a theory of quantum gravity, should relate the physical content of the Universe to its global shape and size. Jürgen Ehlers [private communication] pointed out that in this sense, one could say that general relativity, as a theory relating gravity to geometry, is incomplete, so that while the global geometry of the Universe is independent of the present theory of general relativity, it could be said to be constrained by a more complete form of general relativity, in a theory yet to be found and/or agreed upon.

1.2 Observational detection: topological lensing

Just as general relativity and local perturbations in geometry lead to the observable phenomenon of *gravitational* lensing, whatever theory extends general relativity, yielding constraints on global geometry, would reveal itself by *topological* lensing [18].

This is the basic principle common to (nearly) all suggested techniques of detecting global geometry. (See 2.B.i and 2.B.ii of [15] for the only techniques known to this

author which are independent of this principle.) Just as gravitational lensing is caused by multiple geodesics from an object to the observer, topological lensing would also be caused by multiple geodesics to the observer. In the former case, the geodesics only differ very slightly, over a small portion of their length — due to the gravitational distortion induced by an intervening massive object. But in the latter case, the geodesics are, in general, of very different lengths and point in very different directions, since they are simply two different ways of crossing the Universe between two points.

The “object” observed may either be a gravitationally collapsed, luminous astrophysical object (methods A.i listed in [15]), or a “patch” of photon emitting plasma seen as a fluctuation in the cosmic microwave background (methods A.ii listed in [15]). Due to the pioneering stage of this research, terminology and approaches to classifying the different methods of applying of this principle still vary somewhat.

2 A non-exhaustive list of recommended reading on background mathematics (geometry/topology), (lack of) physical theory and observational strategies

Apart from articles based on workshops in [17] and [1], the following may help guide the reader through the rapidly expanding literature.

2.1 Geometry/topology

For the background mathematics (and some comments on observational strategies), [8] is recommended, but has been complemented recently by a thorough article on multiply connected spherical spaces [7], which have become relevant due to increasing evidence that the observable Universe is approximately flat, just as the surface of the Earth is nearly flat. The curvature radius R_C of a section of the Earth’s surface of radius $R_H \sim 6400$ km or smaller satisfies $R_C \gtrsim R_H$; similarly, the curvature radius R_C of the observable Universe is estimated in comoving units as $R_C \gtrsim R_H \approx 10 h^{-1}$ Gpc, where R_H is the horizon radius of the Universe and the *local* cosmological parameters, Ω_m and Ω_Λ , the density parameter and the cosmological constant respectively, are ($\Omega_m = 0.3, \Omega_\Lambda = 0.7$).

2.2 (Lack of) physical theory

For a shorter description of the background mathematics, but also some references to the beginnings of theoretical work which could be useful for a physical theory of global geometry, [10] is recommended. (This also includes a short historical introduction.) Ideas on topological evolution during the quantum epoch [5] and a diverse range of physical approaches in §VI of [6] are just a few examples of theoretical work.

2.3 Observational strategies

Definitions of the more formal terms corresponding to “shape” and “size”, e.g. the “3-manifold of the spatial hypersurface at constant cosmological time” for “shape”, and the “injectivity diameter” (twice the injectivity radius) for “size”, are illustrated in fig. 10, §5.1 of [10], using the terminology of [3].

Most of the empirical work has (understandably) been done independently of any physical theory of global geometry, i.e. has just assumed a standard Friedmann-Lemaître-Robertson-Walker metric. For a rapid introduction to this approach, newcomers to the field might want to skip straight to §5.1 of [10], and fill in later on the fuller mathematical and historical background.

For an overview and brief description of how the principle of multiple imaging is applied in practice, (i) to collapsed objects spread through three-dimensional comoving space and (ii) to the cosmic microwave background, which would uniquely be in two-dimensional comoving space if the cosmological constant were zero and the density parameter unity, see [12]. Although several methodical developments and observational analyses have been carried out by various groups since [10], the observational constraints on the size of the Universe, i.e. on the injectivity diameter, remain essentially unchanged from the scales listed for the different approaches in table 2 of §5.2 of [10], i.e. $2r_{\text{inj}} \gtrsim 1 h^{-1} \text{ Gpc}$.

The history of the search for exoplanets in the 1990's suggests that the inclusion of a “reasonable” but uncertain theoretical assumption (that a planet massive enough to be detectable could not occur close to its parent star, with an orbital period of only a few days) may lead to ignorance of an astrophysical discovery present in existing observational data. Nevertheless, as a strategical choice it is valid, as long as strong claims are not made.

This is the case in most of the cosmic microwave background analyses for cosmic topology, listed as A.ii.3 in [15].

A brief discussion of what the assumptions are and why they limit the generality of the conclusions of those analyses is provided in §1.2 of [11].

A new major review on cosmic topology, mostly focussing on cosmic microwave background methods, is [9].

For the more observationally minded, two applications of approach A.i.3, using physical characteristics of individual objects in order to detect candidate topologically lensed images, published more recently than refs [12] and [15], are presented here as illustrations of some of the more direct observational approaches possible.

3 GAIA: the Milky Way as the ultimate extragalactic source

By the end of the decade, it is planned to launch a satellite, GAIA, which will make parallax measurements of a billion stars in the Milky Way. This should lead to unprecedented understanding of how the Milky Way formed, and should make it possible to estimate dates of important past events in the history of the Galaxy, including merger events with small neighbouring galaxies, and major events of “nuclear” activity, i.e. of times when our galaxy would have appeared as a quasar or other Active Galactic Nucleus galaxy (AGN) if seen from afar. (In fact, evidence is mounting that mergers and AGN phases probably coincide.)

Since AGNs are typically seen to high redshifts, e.g. $z \sim 2$, the precise dating of past AGN events of the Galaxy would indicate narrow bands of redshift in which topologically lensed images of our Galaxy would be seen. This could lead either to detection of candidate generators (a generator is a path joining topologically lensed

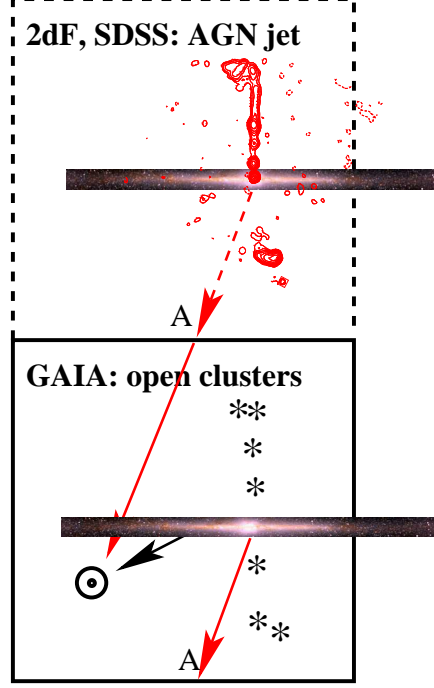


Figure 1: For explanatory purposes, an exaggeratedly small toroidal universe, about 16kpc in size, is shown. As explained in [12], a two-dimensional, flat global universe can be thought of either as (i), a 2-torus placed in ordinary Euclidean 3-space, but then given an intrinsically flat metric, (ii) a “cut-open” 2-torus, i.e. a rectangle of which opposite sides are identified with one another, (called the “fundamental polyhedron”) or (iii) a tiling of the Euclidean 2-plane by multiple copies of the rectangle (called the “universal covering space”). The solid outline (lower square) includes the entire physical universe (fundamental polyhedron). Method (ii) of thinking of the space can be applied by ignoring everything outside this solid outline. The dark arrow shows the geodesic from the Galactic Centre to the observer at the Sun. The gray arrow shows the (long time delay) loop around the universe, where the light from the Galactic Centre takes a lot longer to arrive at the observer. Because the time delay is much bigger, it may correspond to the delay calculated from GAIA data for a “high” redshift (early epoch), extragalactic, AGN phase image of the Galaxy. This is illustrated by extending to the dotted outline (upper square), showing a topological image of the universe, in apparent space, i.e thinking of the Universe as a tiling (iii). The AGN phase is schematically shown by a double lobe radio jet $\sim 10\text{kpc}$ in full length. Returning to the lower half of the figure, open star clusters are shown by asterisks following the shape of the “high” redshift jet. These represent star clusters formed during the AGN phase, which would be observable “today” by GAIA. In reality, they would no longer occupy the “vertical” axis of the Galaxy; they would probably have been through several orbits since the AGN phase during which they formed.

images, corresponding to a single “loop around” the Universe), or to increased confidence in lower limits to $2r_{\text{inj}}$ (apart from caveats due to AGNs being missed in the plane of the Galaxy).

Fig. 1 illustrates the observational situation, and also shows an example of some of the basic elements of global geometry in a Friedmann-Lemaître-Robertson-Walker Universe. For developing one’s basic intuition of observational cosmic topology, it is recommended to be able to switch between thinking in modes (i), (ii) and (iii) as listed in the figure and in, e.g. [12]. Mode (i) is probably the most intuitive for the 2-dimensional case, but difficult for 3-dimensional space; mode (ii) is probably easiest for thinking of the physics; and mode (iii) is generally best for analysing observations.

So GAIA might just possibly turn out to be a powerful probe for observational cosmology. See [19] for a recent discussion of searching for high redshift images of the Galaxy, and [13] for brief comments on the relevance of GAIA.

4 Similar morphologies of radio-loud active galactic nuclei (AGNs)

Although topologically lensed pairs of images would in general occur at very different redshifts, there are certain cases where the redshifts can be very nearly equal. The “matched circles principle” ([2, 4]; see fig. 2 of [12] for an explanation of the principle and fig. 3 of [12] for examples of matching and non-matching circles in 4-year COBE data), adapted in the obvious way for an arbitrary sub-horizon redshift, defines these cases.

A striking coincidence in the morphologies of two radio-loud, double-lobed, compact steep spectrum AGNs, 3C186 and 4C+36.21, yields a good illustration of the falsifiability of specific candidate generators of the global geometry of the Universe [14].

The redshift of 3C186 is known: $z = 1.063$, but the redshift of 4C+36.21 is unknown. These two images could only be two topological images of a single RLAGN if the redshift of 4C+36.21 were to lie in the very small interval which gives a physically reasonable expansion speed for the jet (positive and not slower than about $0.01c$).

4C+36.21 is seen with a linear size in proper units of about $1.6 h^{-1} \text{ kpc}$, and 3C186 is $6.5 h^{-1} \text{ kpc}$ in size, so that the 4C+36.21 image must be just slightly earlier in time than 3C186, but not too much earlier, if the identity hypothesis were to be correct.

The measured redshift of 4C+36.21 would have to lie in the very narrow range $1.0630 < z \lesssim 1.0635$.

A spectroscopic estimate of the redshift of 4C+36.21 is planned. A redshift outside of this range would clearly refute the hypothesis.

A redshift within the required range would be exciting, but would require many more observational tests before it could be considered to provide a serious candidate estimate (roughly $1 h^{-1} \text{ Gpc}$) of the size of the Universe.

5 Conclusion

Although the subject of cosmic topology is just over a century old ([16]; it predates general relativity), it was only in the late 1980’s and the 1990’s that significant observational

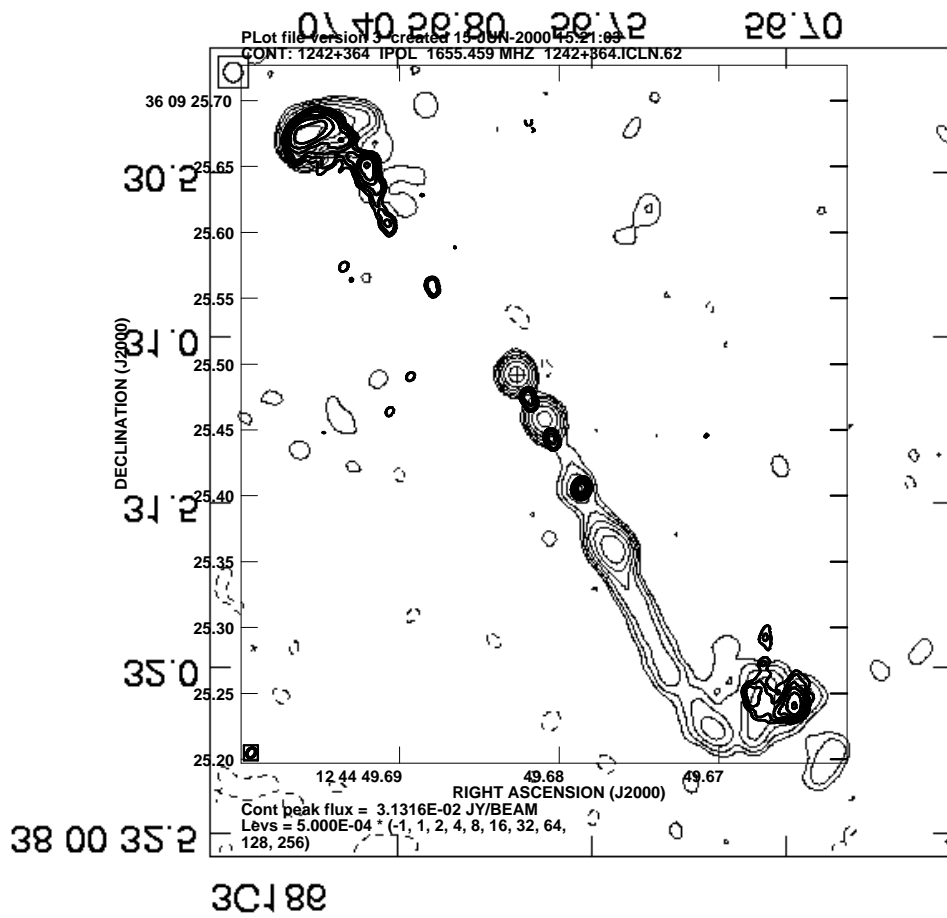


Figure 2: The two strikingly similar radio-loud AGNs. The image of 4C+36.21 (at 18cm) is shown in heavy contours; the image of 3C186 (at 6cm) is shown in light contours and reflected North-South. Either (i) these two images are of physically distinct objects which just happen to show a similar physical process or (ii) they are two topologically lensed images of a single object, separated from one another in comoving space by a “loop around” the Universe.

attention started being paid to the subject. With estimates of the local cosmological parameters finally converging on consistent values, the natural followup is the quest for measuring global cosmological parameters.

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